PERGAMON

International Journal of Impact Engineering 23 (1999) 585–595

INTERNATIONAL JOURNAL OF IMPACT ENGINEERING

www.elsevier.com/locate/ijimpeng

JET PENETRATION INTO LOW DENSITY TARGETS

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Summary — It is well known that the penetration depth of high velocity rods is inversely proportional to the square root of the target density. Hence, it is expected that low density targets are more effective than high density targets in defeating shaped charge jets. However, careful analysis of real jets - with velocity gradients, penetrating targets constrained by fixed areal weight, shows that this is not always the case. Moreover, it was found that there is an optimum value of target density (ρ_{opt}) for maximizing jet erosion. This effect was also verified experimentally.

INTRODUCTION

Consider a short segment of a shaped charge jet of length L and density ρ_j, moving with a constant high velocity, and penetrating a monolithic target of density ρ_t. Neglecting strength effects and assuming a steady-state penetration process the depth of penetration P of this segment is:

$$P = L\left(\frac{\rho_j}{\rho_t}\right)^{1/2} \equiv \frac{L}{\gamma} \tag{1}$$

The mass per unit area of the target necessary to stop this segment is therefore:

$$m = P\,\rho_t = L\,(\rho_j\,\rho_t\,)^{1/2} \tag{2}$$

Hence, the lower the density of the target the lower the weight needed to erode the jet. For example, reducing the density of the target material by a factor of four reduces the mass needed to erode the jet by a factor of 2. The penalty for using low density materials is the low volume efficiency.

This outcome is correct assuming no velocity gradient along the jet, and it is therefore applicable for calculating the penetration depth of very fast long rod projectiles. In reality, shaped charge jets are characterized by the velocities of the tip - V_0 and of the tail - V_{tail}. Real jets are stretching while penetrating a target and their length is therefore not constant in time. The erosion rate and the final stretched length of a jet depend on jet velocity and target characteristics. The penetration of real jets should be described in a way which takes into account the stretching process of the jets.

The spatial distribution of the velocity of most of the jets is almost linear, hence one can assume the existence of a virtual origin (in space and time) from which the jet emerges [1]. Based on these assumptions Allison and Vitali [1] derived the penetration equation for a continuous jet (Case I) as:

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$$P_I = S\left[\left(\frac{V_0}{V_{min}}\right)^{1/\gamma} - 1\right] \qquad \text{Case I} \qquad (3)$$

S is the standoff measured from the virtual origin to the target, and γ is the square root of the target to jet densities, as defined by Eq. (1). V_{min} is the lowest jet velocity that contributes to penetration. The continuously stretching jet breaks into short segments (droplets) at a given time T_b (T breakup). The penetration of a jet that starts penetration as a continuos jet but breaks up before completion of penetration (Case II) can be expressed as:

$$P_{II} = \frac{(1+\gamma)(V_0 T_b)^{1/(\gamma+1)} S^{\gamma/(\gamma+1)} - V_{min} T_b}{\gamma} - S \qquad \text{Case II} \qquad (4)$$

The penetration of a fully particulated jet can be expressed as:

$$P_{III} = \frac{(V_0 - V_{min})T_b}{\gamma} \qquad \text{Case III} \qquad (5)$$

Assuming that the jet particulates all over at the same time - T_b.

The velocity of the tip of the jet V_{out} that emerges from the back of the target with a finite thickness D, can be expressed as (see for example references [2 or 3]):

$$V_{outI} = \frac{V_0}{\left(\frac{D}{S}+1\right)^{\gamma}} \qquad \text{Case I} \qquad (6)$$

$$V_{outII} = \frac{(1+\gamma)(V_0 T_b)^{1/(\gamma+1)} S^{\gamma/(\gamma+1)} - \gamma(S+D)}{T_b} \qquad \text{Case II} \qquad (7)$$

$$V_{outIII} = V_0 - \frac{\gamma D}{T_b} \qquad \text{Case III} \qquad (8)$$

Ignoring all edge effects that might affect these velocities.

Real targets are constrained by a given weight or volume. These constraints should be added to Eq. (4) and solved together. In the following sections we shall analyze the erosion process of a jet penetrating into targets of different areal masses and densities.

WEIGHT CONSTRAINT

Case I

Let us assume that the target has a given mass per unit area - m:

$$m = D\,\rho_t = Const. \qquad (9)$$

Substituting m in Eq. (6) we derive:

$$V_{outl} = \frac{V_0}{\left[\frac{m}{S\rho_t}+1\right]^{\gamma}} \tag{10}$$

Assuming that the jet tail velocity is lower than the minimum velocity necessary for penetrating the target. Plotting V_{outl}/V_0 vs. target density, as shown in Fig. 1, one can see that by reducing the density of the target the velocity of the emerging tip decreases too, up to a certain point, below which the velocity starts to increase with further decreasing of the target density.

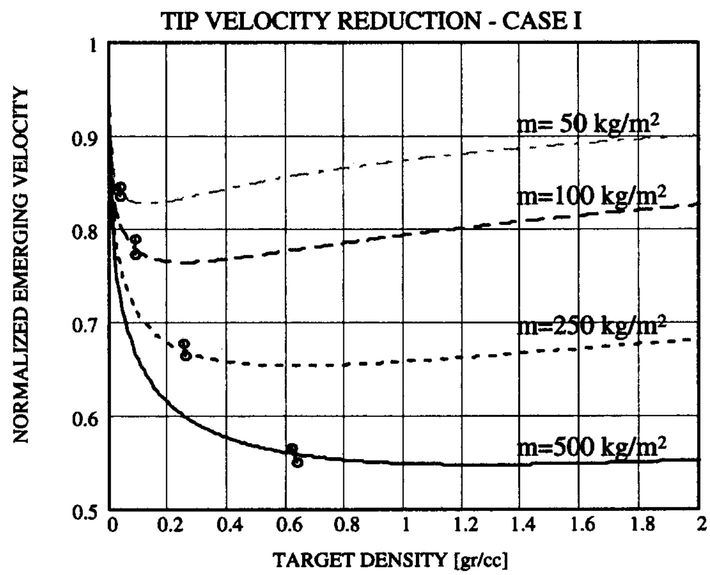

Fig. 1. Relative final jet tip velocity of a continuos copper jet vs. target density, after penetrating a finite thickness target with an areal mass of "m", S=100mm.

The optimal target density is the density of the target at the point where the final jet tip velocity is minimal. For constant mass targets, the lower the density the thicker the target, hence at a certain point the jet will emerge from the back side of the target particulated. To set a limit to the lowest density relevant for Case I, one should note that by considering the target thickness penetrated up to the breakup moment a permissible region for the out coming velocity can be obtained:

$$\frac{S + m/\rho_t}{T_b} < V_{outl} \tag{11}$$

The lines presenting this bound are shown in Fig. 1 for $V_0 = 8$ km/s and $T_b = 200$ μs by the two circles.

In order to find the exact value of the optimal target density - ρ_{opt} as a function of target mass, we need to equate the derivation of V_{out} given by Eq. (10), to zero. ρ_{opt} will be derived by solving the following equation:

$$(x+1)^{[(x+1)/2x]} = e \tag{12}$$

where:

$$x = \frac{m}{S\rho_{opt}} \tag{13}$$

The approximate solution for Eq. (12) is: $a \equiv x \approx 3.92$, and the optimal target density is:

$$\rho_{opt} = \frac{m}{a\,S} \tag{14}$$

Hence, the optimal target density is directly proportional to the target areal weight, and inversely proportional to the stand-off as shown in Fig. 2. The heavier the target the greater the optimal density for defeating the jet. S is usually proportional to the diameter of the charge, consequently the greater the charge the lower the optimal density, for the same target mass.

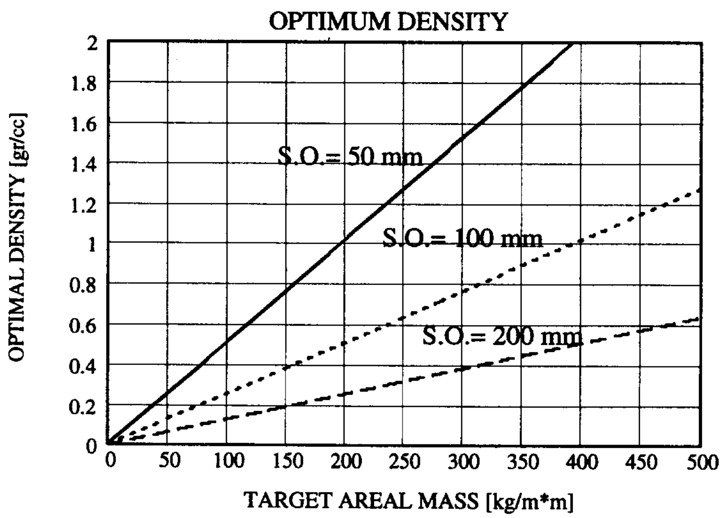

Fig. 2. The optimal target density as a function of target areal mass.

The velocity of the tip of the out coming jet decreases as target mass increases, and may get to a point where all the jet will be fully consumed. Since additional mass will reduce target efficiency, it is important to find the minimum mass required to stop a given charge. The minimum target weight needed to erode this jet - m_{min} - will be found by substituting ρ_{opt} into Eq. (10), and solving it. By doing so we get:

$$m_{min} = \left[\left(\frac{V_0}{V_{min}}\right)^{1/\gamma} - 1\right] S\,\rho_{opt} \tag{15}$$

By substituting the optimal value of the target density from Eq. (14) we get:

$$\rho_{opt} = \rho_j \left[\frac{Ln(V_0 / V_{min})}{Ln(a+1)} \right]^2 \tag{16}$$

The optimal target density needed to erode a continuous jet is shown in Fig. 3. It should be noted that in reality V_{min}, (designated as the cutoff velocity) depends mainly on the yield point and the density of the target and on the alignment and diameter of the jet. The optimal target density needed to erode real jets, with a velocity ratio between 2.5 and 3 is about 3 gr/cc to 4.25 gr/cc. RHA targets are best for eroding jets with tip to cutoff velocity ratio of 4.5, like copper jets with tip velocity of about 13 km/s for example (if it exists). For eroding jets with higher densities higher targets densities should be used in accordance with Eq. (16).

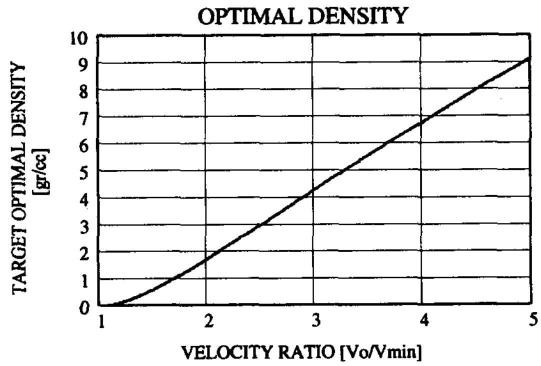

Fig. 3. Optimal target density needed to erode a copper jet.

Using Figs. 2 and 3 one can find out the characteristics of the best target to stop a given jet. Assume, for example, that the jet velocity ratio needed to be stopped is 2, the standoff is 100 mm and that the jet tip velocity is 8 km/s. Hence the target optimal density should be about 1.7 gr/cc, and the target areal mass should be about 660 kg/m^2. This argument is valid for Case I penetration for which the breakup time is longer than 120 μs. If the breakup time is shorter the jet will penetrate this target and emerge particulated.

The dependency of V_{out} on the standoff, after penetrating 100 mm thick targets of various areal masses, is plotted in Fig. 4. It is clearly seen that a target positioned at a very close standoff is very efficient. Scaled up charges operate at longer standoffs, and their penetration capability increase proportionally to the increase in their diameter. Hence, to obtain the same defeating efficiency, the target mass should be increased proportionally, without change in the target density, as can be seen from Eq. (13).

The densities of the targets in four of the curves in Fig. 4 are calculated assuming constant target thickness. In the fifth bottom curve the velocity reduction was calculated differently by assuming optimal density for the target weighing 500 kg/m^2. It is interesting to note that the emerging velocity of the last curve is always lower than that of the constant thickness curve. In addition, according to Eq. (14) the density of the target with constant thickness is higher than the optimal density for long standoffs, but is lower for the short ones.

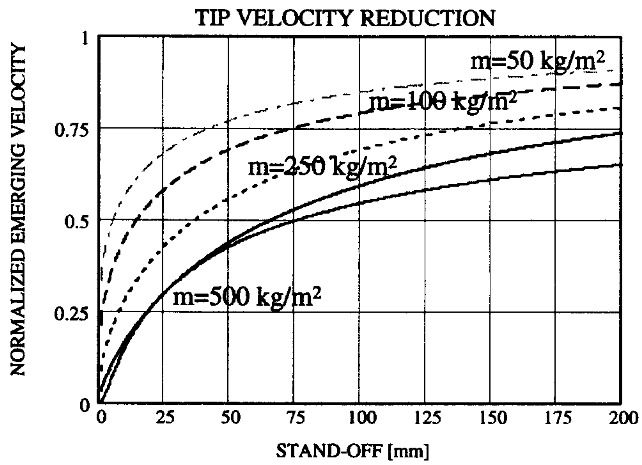

Fig. 4: Relative final jet tip velocity, of a continuous copper jet, vs. stand-off, after penetrating 100 mm thick targets of various areal mass.

Case II

The analysis of Case II can be done in the same way it was done for Case I. Using Eq. (7) the calculated jet tip velocity is plotted as a function of target density, as shown in Fig. 5, for S=300 mm.

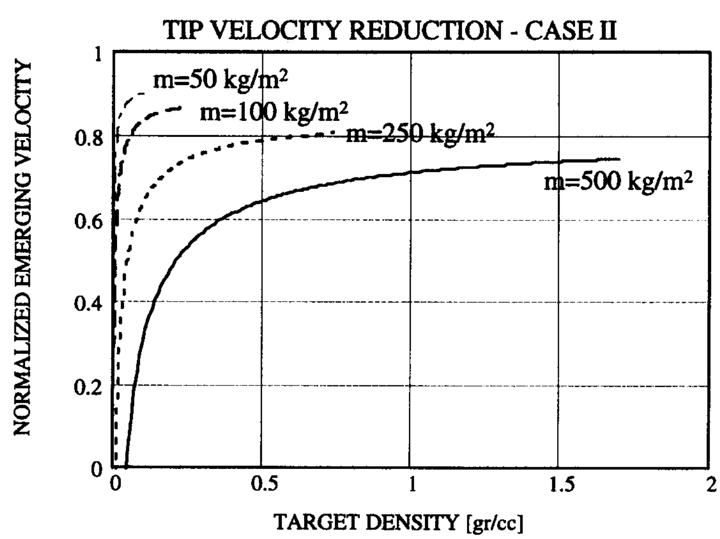

Fig. 5. Relative final jet tip velocity vs. target density for Case II, Vo=8 km/s, T_b = 100 μs.

The curves in Fig. 5 are valid only for density ranges in which the target is thick enough to ensure jet particulation while penetrating - Case II.

To summarize this case, it is clear that for a given target mass, the lower the density of the target the lower the velocity of the tip of the jet that will emerge from the back side of the target.

Case III

Examining Eq. (8), one can see that the velocity of the tip of a fully particulated jet, emerging from the back of a target of areal mass *m* is:

$$V_{outIII} = V_0 - \frac{m / T_b}{(\rho_j \rho_t)^{1/2}} \tag{17}$$

Hence, using a lower target density the out coming jet tip velocity will decrease too. Therefore, for this case, targets made of low density materials are more effective. The reason for this result is the assumption that each particle is moving with a constant velocity and the penetration depth can be calculated using Eq. (1). No bounds on the density of the target are imposed in this case since it is assumed that the jet hits the target after being fully particulated. The final jet tip velocity as a function of target density, for various targets areal mass is shown in Fig. 6.

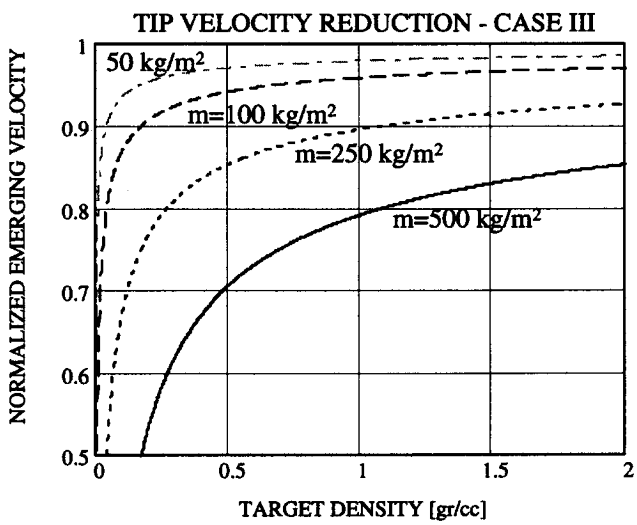

Fig. 6. Relative final velocity vs. target density for Case III, Vo=8 km/s, T_b= 100 μs.

Since the total jet penetration is a function of the lowest effective jet velocity V_{min} that depends on target strength, it is recommended to use low density materials with high yield strength (like ceramics and titanium).

VOLUME CONSTRAINT

If the volume of the target has to be kept constant, then the velocity of tip of the jet that will emerge after penetrating the target will decrease with increasing target density for all three cases, as can be seen from Eqs. (6)-(8).

EXPERIMENTAL SETUP AND RESULTS

The experimental setup is shown in Fig. 7. A precision shaped charge (SC) (84 mm diameter, 60°, point initiated), was used in most of the tests. Different targets were placed 200 mm ahead of the SC. The targets were made of various types of materials and had different thicknesses. The jet emerging from the target was examined using two flashes of X-ray and the residual penetration in RHA witness plates was measured.

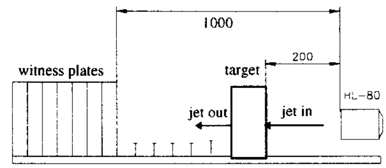

Fig. 7. Experimental Setup

The first test was performed without a target in order to characterize the jet. It was found that the velocity of the tip of the jet is V_0 = 6.95±0.05 km/s, and the tail velocity is V_{tail} =1.97±0.07 km/s. The position of the virtual origin was calculated to be 30 mm away from the charge cone base, towards the cone apex. Eleven additional tests were performed using various targets, as presented in Table 1.

Table 1. Experimental results

Test	*Target*		*Density [gr/cc]*	*Thickness [mm]*	*Emerging Jet Velocity & Time [km/s]* V_{out}	*[μs]* T_{out}
1	RHA	(+)	7.82	40	6.02	47
2	RHA	(+)	7.82	80	5.20	62
3	RHA	(+)	7.82	203	3.65	113
4	RHA	(+)	7.82	320	2.53	195
5	Mild Steel	(o)	7.83	80.5	5.22	63
6	Mild Steel	(o)	7.83	206	3.63	110
7	Aluminum	(◊)	2.75	232	4.75	98
8	Aluminum	(◊)	2.75	232	4.65	96
8	Aluminum	(◊)	2.75	115	5.57	65
10	Polyethylene	(□)	0.945	494	4.62	156
11	Water	(x)	1	573	4.44	181

RHA stands for rolled homogeneous armor steel and the aluminum used was 6061T6Al. The copper jet density is: ρ_j =8.94 gr/cc. The two X-Ray exposures enable us to measure the new tip velocity V_{out}, and the time the jet emerged from the back of the target (measured from the virtual time).

In addition we were able to examine whether this jet is still continuos or particulated.

DISCUSSION

Direct validation of the relation between target mass efficiency and target density is somewhat complicated. The penetration of a jet into a semi-infinite target usually involves penetration of continuous and particulated jet (Case II). The breakup time and the cut-off velocity are not well defined parameters, and they may vary from one charge to another. Hence, many parameters may have to be adjusted to fit the theoretical curve to the experimental data. To avoid these kind of difficulties we chose to validate our observation indirectly by measuring the out-coming velocity of a well defined jet after penetrating a given target and comparing the results with the theoretical values.

Numerical simulations of the penetration process of fast rods and stretching jets into various targets show [5] that the penetration velocity is very close to the one expected by the simple steady-state theory. The initial penetration velocity is higher than expected, in accordance with the planar shock loading of the target at this stage. Soon after the rarefactions from the free boundary reach the axis of symmetry, typically less than one microsecond, the penetration velocity drops down, fluctuates somewhat and, as the penetration proceeds, the rate of penetration slowly approaches the steady-state value.

The comparison between the measured velocities of the jets, that emerge from the back side of the targets, and the calculated values, (Eqs. (6) and (7)) are shown in Fig. 8. The calculated lines are for three materials: steel (bottom), aluminum (center), and water (top). For each material we have plotted two lines: one for the continuous jet - Case I (solid line), and one for the mixed mode of penetration- Case II (dashed line).

In all the experiments in which the jets penetrated the targets unbroken, the velocities of the emerging jets measured experimentally are in excellent agreement with those calculated by Eq. (6). In some cases there is a small deviation between the two values. The main reason for this deviation is the break-up process of the jet.

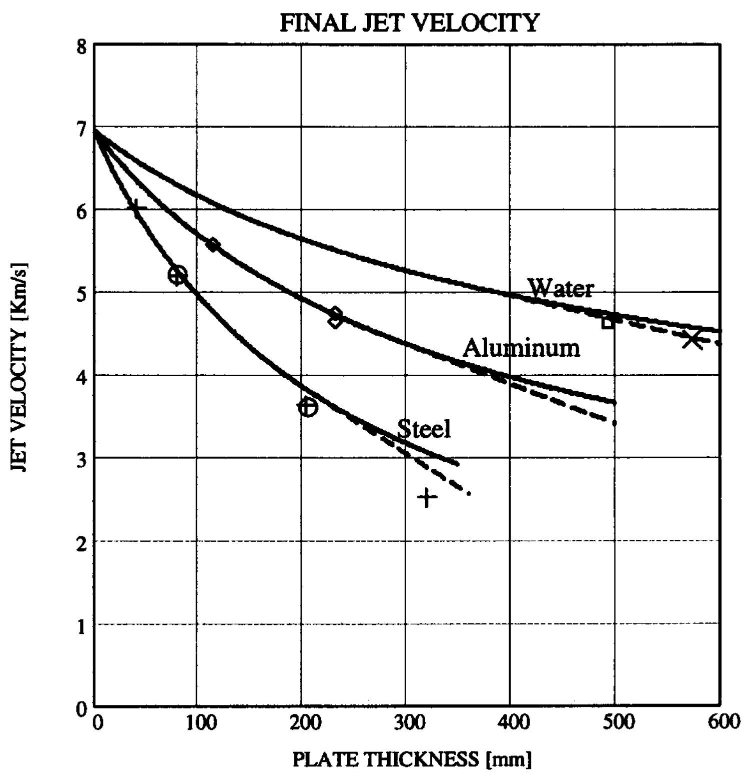

Fig. 8. Jet tip velocity after perforating a finite thickness target. Experimental (symbols: presented in Table 1) vs. calculated results

Although it is commonly assumed that the jet breaks up all over at the same time, this is not exactly true in reality. Some parts of the jet break earlier, as seen for example in ref.[4]. In addition we have to take into consideration that the jet is ductile and the break-up, a necking process, is not instantaneous. Breaking can occur over a period of ten to twenty microseconds (or even more), depending on the jet diameter, local strain rate and material properties.

The breakup time of the jet examined varies along the jet and is about 115 μs at the middle of the jet, as measured from the virtual time. Hence the jets that emerge from the back side of the water and polyethylene targets in tests number 10 and 11, were expected to be particulated, as indeed was observed in the X-Ray radiographs. These experimental data points in Fig. 8 should therefore be compared with the Case II dashed line, and the agreement is indeed very good.

In three tests, using steel targets, the measured velocities are lower than expected. In one of tests the jet is particulated - test number 4 (RHA), and in the other two the emerging jet is continuous - tests number 3 (RHA) & 6 (MS), as seen in Fig. 9. A possible explanation for this phenomena is the strength of the steel, that decelerate the penetration rate. Another explanation for this discrepancy is an early break-up of jet.

Break-up is usually defined as the time the jet starts to particulate but it should be defined as the time the accumulated length of the jet segments ceases to increase. This time is earlier than the actual break-up time based on the old classical definition. This topic will be studied in detail in the near future.

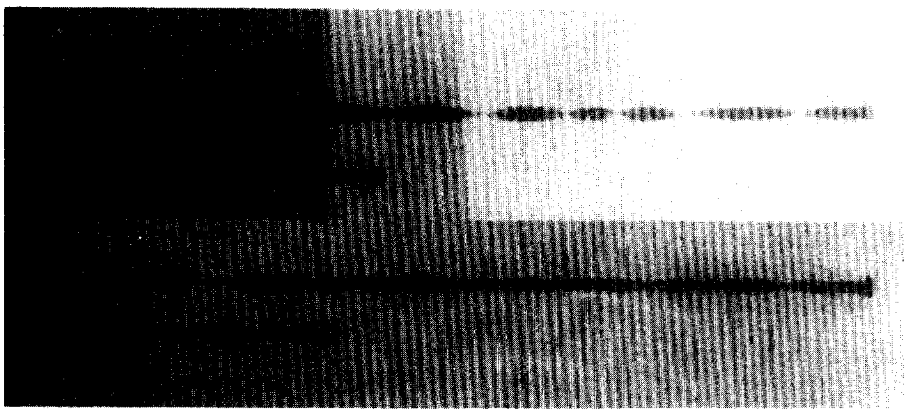

Fig. 9. X-Ray radiographs of the jets emerging from the back side of a 203 mm RHA target (top) and 206 mm mild steel target (bottom), at 140 μs and 170 μs.

To examine experimentally the efficiency of very low density targets (lower than the optimal one), we considered the penetration of jets into air. According to Backofen [6], the tip of jet is slowed and eroded while moving in air. Taking into account the compressibility of air, the erosion length should be about 30% smaller compared with the incompressible model (Eq. (1)). It is noted that this theoretical model is based on the assumption of steady flow. This assumption seems well justified for air, but does not hold for the jet tip erosion flow unless the ratio between the eroded length and jet diameter is considerably larger than unity. The reasoning behind this argument is that the steady flow established about the jet-air interface is a two-dimensional one (like a fountainhead). The time to establish such steady flow is proportional to its characteristic lateral dimension, i.e., the jet diameter. It would therefore take a run of a large distance before the jet tip erosion would be given in terms of the steady flow (one-dimensional) theory. Therefore, to test erosion in air small charges should be used.

Using small charges, we found that over a distance of about one meter in air (about 1.3 kg/m^2) the missing length of jet particles along the jet path (some of which are eroded) should be about 12 mm, in accordance with Eq. (1). As can be seen in Fig. 10, the tip of the jet after one meter distance of flight is spread into many small particles (circled) that amount to about 4 mm jet length. Hence the exact eroded length is not well defined in this case, and it is not clear if this experiment validates the incompressible theory. It should be noted that only particles in the front section of the jet were eroded, as was also observed by Held [7].

By neglecting the front spread out particles, the experimental observation confirms the conclusions presented in this paper: targets made of very low density materials are less efficient compared with those made of materials with the optimal density. The compressibility of air (and of other low density targets) changes somewhat the value of the optimal density (especially for jets with small diameters), in favour of lower density materials.

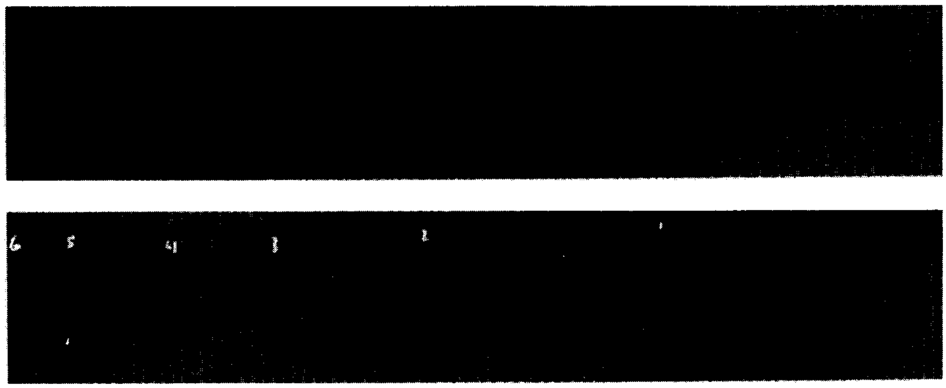

Fig. 10. X-Ray radiograph of the front part of a jet at two standoffs in air:
top - 300 mm, bottom - 1300 mm.

CONCLUSIONS

Analyzing the penetration process of a shaped charge jet into targets of constant weight and various densities it was found that there is an optimal target density for eroding continuous jets.

This new concept is complementary to the classical view that low density targets are more effective in defeating shaped charge jets, a view which was found to be true only for particulated jets.

The analysis is based on the assumption that penetration is a steady-state phenomenon. The velocity of the out coming jet was measured and was found to be in a very good agreement with the classical penetration model.

Different values of the optimal density were derived and plotted as a function of various constraints, such as the areal mass and the standoff, and the characteristics of the jet.

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