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Short Communication

Analytical approach to the strain rate effect on the dynamic tensile strength of brittle materials

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ABSTRACT

An explicit mathematical expression for the dynamic load-carrying capacity of brittle materials under dynamic tensile loads is derived based on a kind of structural-temporal failure criterion [1] and the one-dimensional longitudinal plane wave propagation model. It is shown that the dependence of the dynamic load-carrying capacity on the strain rate can be determined only by the static material parameters such as tensile strength, density, incubation time, critical failure length and constitutive constants, which verifies that the well known strain rate effect on material strength can be considered as an structural rather than material behavior, as pointed out by Cotsovos and Pavlović [2] recently. Moreover, it is found that, under constant strain rate, the dynamic load-carrying capacity depends also on the amplitudes of imposed boundary loads, which explains, to a significant extent, the scatter that characterizes the available experimental data. Furthermore, the derived expression can also be used as a foundation of theoretical analyses on other problems involving the strain rate effect such as dynamic size effect, dynamic failure of quasi-brittle materials and composites.

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1. Introduction

As is well known, most of materials can bear higher dynamic loadings without failure than quasi-static ones, which is usually known as the strain rate effect upon strength of material. The strain rate effect is crucial when dealing with problems involving dynamic loadings such as in the fields of penetration mechanics, explosion mechanics and high velocity collision. Over the past decades, a number of research papers, see for examples references [3–8], have been published on both the theoretical and the experimental studies of the strain rate effect for various materials, and some dynamic failure criterions are established. However, up to now, the strain rate effects are still far from to be well understood. On the one hand, most of the dynamic failure criterions are obtained by the curve fitting of experimental data, which may not give results that are applicable in all situations. On the other hand, from the theoretical point of view, the physical mechanism as well as the mathematical expression of the strain rate effects is still unclear. Among those previous studies on the strain rate effect, two commonly accepted viewpoints emerged. One was to say that material failure occurs instantaneously when the local stress reaches a critical stress (the dynamic failure strength of materials); the dynamic failure strength is assumed to be dependent to the local strain rate, and the relation between the dynamic failure strength and the strain rate was taken as an intrinsic property of materials, which can be derived experimentally. This viewpoint leads to the so-called critical dynamic strength criterion, which is a natural extension of the classical maximum stress criterion under quasi-static loadings and has been used widely in many engineering applications. However, it should be stressed that, on the one hand, the dynamic failure strength criterion does not reflect directly time effects on the dynamic response of materials, and time-dependent phenomena such as inertial and wave effects are usually crucial in the problems of dynamic mechanics. On the other hand, the factitious assumption that the dynamic failure strength is dependent on the strain rate may lack the necessary physical basis, which can be partly verified to be problematic by the scatter that characterizes the available experimental data. The other point of view was to say that the dynamic failure strength is not an intrinsic property of materials (and, instead of the dynamic failure strength, a new nomenclature is suggested in this paper termed the dynamic load-carrying capacity to avoid ambiguity), but just a computable characteristic, which can be determined definitely by static material parameters, imposed boundary loadings and structural geometry. Cotsovos and Pavlović [2] investigated the dynamic failure of concrete by the numerical finite element method using a quasi-static constitutive model as well as static material strength, and claimed that the numerical results were consistent with the published experimental data. This second viewpoint attributes the

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strain rate effects to the result of inertial effect and wave propagation, which is a promising model for describing the physical mechanism of the strain rate effects. Of more important significance is that, according to the second viewpoint, dynamic failure of materials can be determined only by the static material parameters, which will be easily used in engineering applications. Certainly, more in-depth theoretical and experimental studies are still needed to affirm the correctness and reality of this viewpoint. Beside the above mentioned work, a structural-temporal failure criterion was suggested recently [1], which is based on the mean impulse acting on the structural components; material failure is governed by the incubation time τ and the critical failure length δ. Both τ and δ are material constants. The concept of the incubation time has been investigated in dynamic fracture mechanics by many researchers [9–12] and that of the critical failure length has become the basis of discrete fracture mechanics developed recently [13].

The aim of this paper is to investigate analytically in more detail the strain rate effect on material strength to affirm the theoretical probability and rationality for taking the dynamic failure strength as a computable characteristic of materials. For simplicity, the model system of one-dimensional longitudinal plane wave propagation under dynamic tensile loads is used. Based on the above-mentioned structural-temporal failure criterion, and through a direct but a little tedious calculus, an explicit mathematical expression for the dynamic load-carrying capacity is derived, which shows that the dynamic load-carrying capacity depends on both the static tensile strength σ_s and the strain rate $\dot{\varepsilon}$, and can be determined completely by static material parameters. Moreover, it is demonstrated that the dynamic load-carrying capacity may be quite different from each other under different imposed boundary loads; associated with different amplitudes of imposed boundary loads, the dynamic load-carrying capacity can take all values which consist of a valued field (which can be termed as the critical load interval) with the static tensile strength as its lower limit. Thus the scatter of available experimental data can be explained qualitatively. Furthermore, it also indicates that, under an incident pulse with the wavelength long enough, material failure can occur when the dynamic load-carrying capacity is equal to the static tensile strength. In other words, for an incident pulse with the wavelength long enough (compared with the incubation time τ), there is not any strain rate effect on the fracture strength of materials.

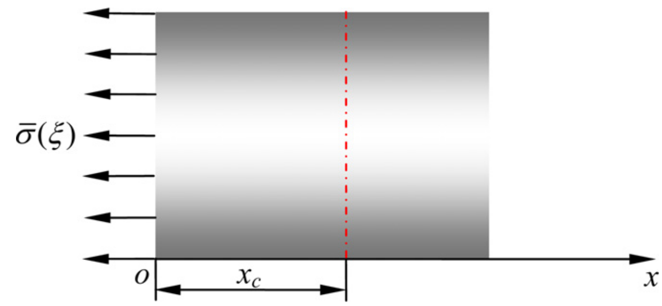

Fig. 1. One-dimensional longitudinal plane wave propagation in a half-space.

2. Dynamic load-carrying capacity

The one-dimensional longitudinal plane wave propagation model used in the paper is shown in Fig. 1, where x_c denotes the location at which the material failure occurs. When a dynamic pulse load acts on the left boundary of the half-space, an incidence linear elastic longitudinal wave is produced and transferred the half-space from the left to the right. The well-known governing differential equation can be written as

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c_L}\frac{\partial^2 u}{\partial t^2} \tag{1}$$

where u is particle displacement; ρ and c_L are the density and the velocity of longitudinal wave of the material occupying the half-space, respectively, $c_L = [(\lambda + 2\mu)/\rho]^2$; λ and μ are the Lamé constants of the material. For the half-space initially stress free and at rest, the initial conditions is

$$u(x,0) = \frac{\partial u(x,0)}{\partial t} = 0 \tag{2}$$

the imposed trapezoid boundary loads (Fig. 2) can be written as

$$\sigma(0,\xi) = \overline{\sigma}(\xi) = \begin{cases} k_1\xi & 0 \le \xi \le \xi_a \\ k_1\xi_a = \sigma_A & \xi_a \le \xi \le \xi_b \\ \sigma_A - k_2(\xi - \xi_b) & \xi_b \le \xi \le \xi_u \end{cases} \tag{3}$$

where ξ is the time parameter on the boundary, and $T = \xi_b - \xi_a$, $\sigma_A - k_2(\xi_u - \xi_b) = 0$.

Under the initial conditions (2) and the imposed boundary condition (3), the solution of the governing Eq. (1) can be derived as [14] (We use here the tensile stress $\overline{\sigma}$ to replace the compressive pressure $-p$ presented in [14])

$$u(x,t) = -2B\int_0^{(t-x)/c_L} \overline{\sigma}(\xi)d\xi \tag{4}$$

where constant $B = c_L/2(\lambda + 2\mu)$, and then, through a simple but a little tedious calculus, the non-zero time-dependent displacement, strain and stress fields in the one-dimensional half-space can be obtained as

$$u(x,t) = \begin{cases} -Bk_1(t - x/c_L)^2 & 0 \le t - x/c_L \le \xi_a \\ B\sigma_A[\xi_a - 2(t - x/c_L)] & \xi_a \le t - x/c_L \le \xi_b \\ B\left\{\sigma_A[\xi_a - 2(t - x/c_L)] + k_2(t - x/c_L - \xi_b)^2\right\} & \xi_b \le t - x/c_L \le \xi_u \\ -2B\sigma_A(T + \xi_u) & \xi_u \le t - x/c_L < \infty \end{cases} \tag{5a}$$

$$\varepsilon(x,t) = \begin{cases} Ck_1(t - x/c_L) & 0 \le t - x/c_L \le \xi_a \\ C\sigma_A & \xi_a \le t - x/c_L \le \xi_b \\ C[\sigma_A - k_2(t - x/c_L - \xi_b)] & \xi_b \le t - x/c_L \le \xi_u \end{cases} \tag{5b}$$

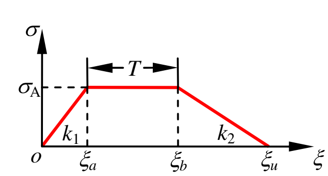

Fig. 2. Boundary condition.

$$\sigma(x,t) = \begin{cases} k_1(t-x/c_L) & 0 \le t-x/c_L \le \xi_a \\ \sigma_A & \xi_a \le t-x/c_L \le \xi_b \\ \sigma_A - k_2(t-x/c_L-\xi_b) & \xi_b \le t-x/c_L \le \xi_u \end{cases} \tag{5c}$$

where $C = 2B/c_L = 1/(\lambda + 2\mu)$.

According to Morozov and Petrov [1], the structural-temporal failure criterion can be written as

$$\int_{t-\tau}^{t} \sigma(x,t)\mathrm{d}t \ge \tau\sigma_s \tag{6a}$$

for intact materials, and

$$\int_{t-\tau}^{t} \mathrm{d}t \int_{x-\delta}^{x} \sigma(x,t)\mathrm{d}x \ge \tau\delta\sigma_s \tag{6b}$$

for a cracked material, where σ_s is the static failure strength of materials. In this paper, we will investigate only intact brittle materials. However, from the viewpoint of the physical mechanism, regardless of intact or cracked materials, material failure is always induced by nucleation and growth of microcracks and hence there should exist an intrinsic critical failure length δ to describe the "quantal [13]" microcrack growth. Accordingly, Eq. (6b) will be used as the failure criterion in the following calculations. Additionally, it is also easy to validate that the result derived by using Eq. (6a) is equivalent to that derived by Eq. (6b) with vanishing δ.

Without loss of generality, material failure is assumed to occur at $x = x_c$ and $t = t_c$. By substituting Eq. (5c) into Eq. (6b), the dynamic load-carrying capacity can be derived, which can be presented as the following three expressions representing respectively three main failure modes depending on the relations between the incubation time τ and the imposed boundary pulse waveform:

a) If $x_c/c_L \le t_c - \tau_L < t_c \le t_a$, we have

$$k_1\left[\left(t_c - \frac{x_c}{c_L}\right) - \frac{1}{2}\left(\tau - \frac{\delta}{c_L}\right)\right] \ge \sigma_s \tag{7}$$

The first term of the left hand side of the inequality is just the dynamic load-carrying capacity σ_c, and the local strain rate can be derived from (5b) with $\dot{\varepsilon} = k_1/(\lambda+2\mu)$, and dynamic load-carrying capacity is obtained:

$$\sigma_c \ge \sigma_s + \frac{\lambda+2\mu}{2}\left(\tau - \frac{\delta}{c_L}\right)\dot{\varepsilon} = \sigma_s + \frac{1}{2}\rho c_L(c_L\tau - \delta)\dot{\varepsilon} \tag{8a}$$

b) If $t_a \le t_c - \tau < t_c \le t_b$, we reach to the maximum tensile stress criterion under static loads:

$$\sigma_c = \sigma_A \ge \sigma_s \tag{8b}$$

c) If $x_c/c_L \le t_c - \tau \le t_a \le t_c \le t_b$, as shown in Fig. 3, we have

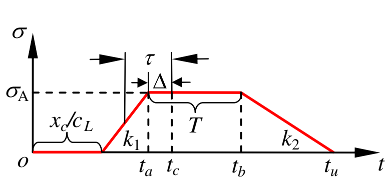

Fig. 3. Temporal curve at $x = x_c$.

$$\sigma_c \ge \sigma_s + \frac{1}{2}\rho c_L\left(1 - \frac{\Delta}{\tau}\right)[c_L(\tau - \Delta) - \delta]\dot{\varepsilon} \tag{8c}$$

where $0 \le \Delta \le \tau$.

3. Discussions and conclusions

In Section 2, the explicit expressions for dynamic load-carrying capacity as functions of the static tensile strength and the strain rate are obtained in the form of Eqs. (8a–c). Some conclusions for the dynamic load-carrying capacity of brittle materials can be drawn out immediately from these analytical results as follows.

Firstly, it is important to distinguish between the concept of dynamic load-carrying capacity and of the (static) strength of materials; the dynamic load-carrying capacity changes with strain rate while the strength does not. The usually termed strain rate effect is actually induced by the change of the load-carrying capacity rather than the intrinsic strength of materials. However, as pointed out by Cotsovos and Pavlović [2], the dynamic load-carrying capacity is never an intrinsic property of materials, which can be determined completely by the imposed boundary conditions and local strain rate with some material constants including the static strength, incubation time, critical failure length, density and other constitutive constants. In a word, the derived results are verified, for brittle materials, that the strain rate effect may be determined by static properties of materials.

Secondly, it can be demonstrated that the dynamic load-carrying capacity will change with different imposed boundary loading pulse. During the propagation of the incident pulse through the failure location x_c, if the amplitude of the pulse $\sigma_A \ge \sigma_c$ and $\tau \le \xi_a = t_a - x_c/c_L$, material failure will occur within the ascending branch of the pulse, and thus the failure is governed by Eq. (8a), which shows that in general $\sigma_c > \sigma_s$ because of the strain rate effect. The upper limit value of σ_c can be easily obtained from Eq. (8a) by assuming that $\tau = \xi_a$ as

$$\sigma_{c\ \max} = \sigma_s + \frac{1}{2}\rho c_L(c_L\xi_a - \delta)\dot{\varepsilon} \tag{9}$$

On the other hand, in the case of $\sigma_A < \sigma_c$ within the whole ascending branch, material failure cannot occur at this stage. Afterwards, if $\sigma_A = \sigma_s$ and $\tau < T$, material failure will occur within the pulse plateau according to Eq. (8b) with the vanishing strain rate, which yields that $\sigma_c = \sigma_A = \sigma_s$ and this is also the lowest limit value of σ_c, i.e., $\sigma_{c\min} = \sigma_s$. Furthermore, it can also be verified that, in other cases where $\sigma_c > \sigma_A > \sigma_s$, the dynamic load-carrying capacity will take all the values in between the two limits, i.e., we have

$$\sigma_s \le \sigma_c \le \sigma_{c\ \max} \tag{10}$$

Therefore, the values of dynamic load-carrying capacity will come into being a range with different incident pulse and incubation time, which may be used to explain qualitatively the scatter that characterizes the available experimental data. Moreover, it should noted that material failure can occur under dynamic load amplitude $\sigma_A = \sigma_s$ only if the pulse width is long enough compared with the incubation time of the material.

Thirdly, the derived results (8a) and (8b) show that the so-called dynamic increasing factor σ_c/σ_s is also a function of both material strength and strain rate rather than that of strain rate only as assumed usually. Moreover, in general, materials with different static strengths should also be associated with quite different incubation times and critical failure lengths. Accordingly, attempting to acquire an available relation between the dynamic increasing factor and strain rate is problematic. In other words, the dynamic increasing factor may not be a reasonable parameter to describe the dynamic response of materials.

Finally, it should be pointed out that, under some special cases, the dynamic load-carrying capacity may decrease with increasing strain rate, as shown by Eqs (8a) and (8b) under $c_L\tau < \delta$, although in general the inequality may not be satisfied by many kinds of materials.

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References

[1] Morozov N, Petrov Y. Dynamic of fracture. Springer; 2000.
[2] Cotsovos DM, Pavlović MN. Numerical investigation of concrete subjected to high rates of uniaxial tensile loading. International Journal of Impact Engineering 2008;35:319–35.
[3] Grady DE. The spall strength of condensed matter. Journal of the Mechanics and Physics of Solids 1988;36:353–84.
[4] Dekel E, Eliezer S, Henis Z, Moshe E, Ludmirsky A, Goldberg IB. Spallation model for the high strain rates range. Journal of Applied Physics 1998;84: 4851–8.
[5] Murray NH, Bourne NK, Rosenberg Z, Field JE. The spall strength of alumina ceramics. Journal of Applied Physics 1998;84:734–8.
[6] Díaz-Rubio FG, Pérez JR, Gálvez VS. The spalling of long bars as a reliable method of measuring the dynamic tensile strength of ceramics. International Journal of Impact Engineering 2002;27:161–77.
[7] Brara A, Klepaczko JR. Experimental characterization of concrete in dynamic tension. Mechanics of Materials 2006;38:253–67.
[8] Weerheijm J, van Doormaal JCAM. Tensile failure of concrete at high loading rates: new test data on strength and fracture energy from instrumented spalling tests. International Journal of Impact Engineering 2007;34:609–26.
[9] Steverding B, Lehnigk SH. Response of cracks to impact. Journal of Applied Physics 1970;41:2096–9.
[10] Kalthoff JF, Shockey DA. Instability of cracks under impulse loads. Journal of Applied Physics 1977;48:986–93.
[11] Ravi-Chandar K, Knauss WG. An experimental investigation into dynamic fracture: 1. crack initiation and arrest. International Journal of Fracture 1984;25:247–62.
[12] Shockey DA, Erlich DC, Kalthoff JF, Homma H. Short-pulse fracture mechanics. Engineering Fracture Mechanics 1986;23:311–9.
[13] Taylor D. The theory of critical distances. Engineering Fracture Mechanics 2008;75:1696–705.
[14] Achenbach JD. Wave propagation in elastic solids. North-Holland Publishing Company; 1973.